A-Priori Accuracy ff 1D Coordinates in the Network of Combined Levelling

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Abstract

Vertical positioning means determination of a vertical position (height) of a point related to another, i.e. determination of height differences. In engineering geodesy, the height differences are determined by a combination of the geometrical and trigonometric levelling. Establishing of 1D geodetic network is performed according to a measurement plan, created by accuracy demands. It consists of: selection of instruments and accessories, defining the accuracy conditions, defining monitoring of data and control of measurements. In this paper we will present the activities on preparation, realisation and processing of measuring quantities, in order to determine the height differences and point heights of the operative polygon within the building the highway corridor Vc, segment Počitelj-Bijača, sub-segment Zvirovići-Bijača. Accuracy estimation and the choice of the measuring method in the combined levelling network were done following accuracy demands of height elements which are a part of the design estimation.

Key words: Levelling, accuracy demands, tolerance

1 INTRODUCTION

When designing engineering structures, the establishment of a 1D network on the construction site is of crucial importance within their vertical spatial localisation. The accuracy and selection of measurement methods, in such a network is calculated based on the required accuracy of height elements included in the calculation of the design. After this establishment, 1D network becomes the basis of all classical measuring and marking heights during the construction and exploitation in the zone of the structure. The project of 1D network on the construction site must take into account elements that affect its shape, accuracy and method of stabilisation. They are primarily elements of the construction site position, the structure position (over ground, underground), type of structure, land quality and other specific features of the structure (Begović, A, 1990.).

2 RESEARCH OBJECTIVE AND METHODS

1D network designed for special needs consists of a basic network of I order, and networks of II and III order if needed. The levelling network project should include: review plan or a map with all the information about the existing geodetic networks of the area included in the process of setting 1D network; method of stabilisation; measurement plan, which is developed according to the given accuracy and cost analysis.

Review plan or map of geodetic network is made in the scale of 1:2500 to 1:10 000, depending on the size of the construction site. Traverses of the levelling network are designed along main communications in the shape of a polygon or a system of nodal points (Mihailović, K, 1978.).

Points of 1D geodetic network (benchmarks) are stabilised so that they can be easily used in the construction and monitoring of the structure. Benchmarks can be of a different design, but it is very important that they are not destroyed or damaged. On large construction sites damages are frequent due to earthworks, transportation, and other circumstances, so it is necessary to predict control benchmarks placed outside the zone of construction works on the geologically stable terrain. They can be built in horizontally or vertically and made of cast iron or aluminium.

2.1 MEASUREMENT PLAN

Detailed elaboration of the measuring method is carried out on the basis of the defined accuracy of 1D coordinates of geodetic network points. The required geodetic measurements are carried out based on the measuring plan, which is made by a given accuracy and includes: selection of instruments and accessories, defining terms of accuracy and defining data for measurement monitoring and control.

2.1.1 A-priori accuracy assessment of 1D coordinates

In order to investigate whether using certain methods of measurement the required accuracy may be achieved and under which circumstances, it is previously necessary to perform the analysis of the method. This analysis should determine the sources of errors in measuring, the accuracy with which it is necessary to perform certain activities in the measurement process, as well as what are the options for eliminating or reducing the impact of systematic errors with the working method or by making appropriate corrections. A-priori assessment of accuracy is performed for a specific type of levelling based on determining the values of some elementary errors. The expressions for the a-priori assessment of accuracy of height differences include:

- assessment of accuracy of geometric levelling,
- assessment of accuracy of trigonometric levelling.

A-priori assessment of accuracy of geometric levelling is performed starting from the individual sources of errors in the expression for the mean squared error of height difference at the station (Mrkić, R., 1991., p.332)):

$$\sigma_{\Delta h}^2 = \frac{1}{2n}\left(\sigma_{DR}^2 + d^2\left(\frac{2\sigma_r^2 + \sigma_o^2}{\rho''^2}\right)\right) + \Delta h^2\sigma_M^2 + 2\sigma_z^2 . \qquad (1)$$

The height difference between two benchmarks is calculated as: $\Delta H = \Delta h_1 + \ldots + \Delta h_{n_h}$. For a homogeneous accuracy: $\sigma_{\Delta h_1} = \ldots = \sigma_{\Delta h_{n_h}}$ mean squared error of height difference between two benchmarks from n_h stations, is calculated according to the expression:

$$\sigma_{\Delta H}^2 = n_h \sigma_{\Delta h}^2 . \tag{2}$$

If we start from approximate values of certain sources of errors: $\sigma_{DR} = \pm 20$ μm - random error of division of rod; $\sigma_r = \pm 0.05''$ - random refraction; $\sigma_o = \pm 0.5''$ - random error in rod readings; $\sigma_M = \pm 7$ μm - rod-pair scale error; $\sigma_z = \pm 29$ μm - rounded readings error (Mrkić, 1991 and Činklović, 1973.), mean squared error of height difference at the station of one levelling side (measured twice $n = 2$) for different lengths of line of sight D , with a maximum height difference at the station Δh = 1 m is calculated in Table 1.

Table 1 Values of height difference errors

D (m)	10	25	50
	$\sigma_{\Delta h}$ (mm)		
n=2	0.04	0.05	0.07
n_h	$\sigma_{\Delta H}$ (mm)		
5	0.10	0.12	0.17
10	0.14	0.17	0.24
20	0.20	0.24	0.33
30	0.24	0.29	0.41
50	0.31	0.37	0.53

Assessment of accuracy of trigonometric levelling is performed starting from the expression used for determination of height difference based on the observed zenith distances (Mihailović, 1978):

$$\Delta h_A^B = D\,ctgz_A + i_A - l_B + (1 - k_A)\frac{D^2}{2r} + D\,ctgz_A \frac{H_m}{r} . \tag{3}$$

If due to the curvature of the earth's surface $\frac{D^2}{2r}$ and point heights $D\,ctgz_A \frac{H_m}{r}$ members are considered negligible for the lengths less than 1000 m, the mean squared error of height difference is found by applying the principle of error transfer on the expression (3)

$$\sigma_{\Delta h}^2 = ctg^2 z \cdot \sigma_D^2 + \frac{D^2}{\sin^4 z}\sigma_z^2 + \sigma_i^2 + \sigma_l^2 + \frac{D^4}{4r^2}\sigma_k^2 , \tag{4}$$

where: $\sigma_{\Delta h}$ - standard deviation of height difference, σ_z - standard deviation of the measured zenith distance, σ_D - standard deviation of the measured length, σ_k - standard deviation of the refraction coefficient, σ_i - standard deviation of the measured instrument height, σ_l - standard deviation of the measured signal height.

The impacts of standard deviations σ_D, σ_i and σ_l do not depend on the length *D*. The impacts of standard deviations σ_z and σ_k depend on the length *D* and are larger for longer distances. The expression (4) can be simplified if the first member is left out, which is negligible, it remains that the accuracy of height difference determination by using trigonometric levelling:

$$\sigma_{\Delta h}^2 = \frac{D^2}{\sin^4 z}\sigma_z^2 + \sigma_i^2 + \sigma_l^2 + \frac{D^4}{4r^2}\sigma_k^2 \tag{5}$$

If the height difference ΔH is determined on the basis of reciprocal measurement Δh_1 and Δh_2 then the mean squared error of reciprocally determined height difference is:

$$\sigma_{\Delta H}^2 = \frac{1}{4}\sigma_{\Delta h_1}^2 + \frac{1}{4}\sigma_{\Delta h_2}^2 \qquad (6)$$

or for homogenous accuracy $\sigma_{\Delta h_1} = \sigma_{\Delta h_2} = \sigma_{\Delta h}$ and with simultaneously determined height differences the impact of refraction can be neglected, and thus we have:

$$\sigma_{\Delta H}^2 = \frac{1}{2}\sigma_{\Delta h}^2 = \frac{1}{2}\frac{D^2}{\sin^4 z}\sigma_z^2 + \frac{1}{2}\sigma_i^2 + \frac{1}{2}\sigma_l^2 \qquad (7)$$

The question is: Which instrument should be used for measuring zenith distances, knowing that it is necessary to determine height difference with accuracy $\sigma_{\Delta H} = 5$ mm and where $\sigma_{i,l} = 3$ mm.

Table 2 Values zenith distance errors

D (m)	100	300	500	550
σ_z (”)	11.67	3.89	2.33	2.12

Table 2 shows that the impact of error σ_z depends on length *D*, and is larger for longer distances, which must be taken into account in the selection of instruments and accessories.

2.1.2 Weights of height difference

Weights of height difference are determined starting from the expression:

$$P_{\Delta H} = \frac{k}{\sigma_{\Delta H}^2}, \qquad (8)$$

where k is arbitrary constant, and $\sigma_{\Delta H}^2$ is mean squared error of a certain height difference in the traverse of geometric levelling according to the expression 2 and for trigonometric levelling according to the expression 7.

2.2 ADJUSTMENT 1D COORDINATES OF THE LEVELLING NETWORK

Determinations of 1D coordinates are adjust by the method of least squares. The measured values in the network are: height differences between the points of operating polygon of the 1D network. Height differences of basic 1D network, with a mean error of less than 0.3 mm/km is determined by using the geometric levelling, and height differences of network II and III for which accuracy of the order of 5 mm is expected can be determined using trigonometric levelling for the lengths of less than 550 m. The measured values in the adjustment are independent values.

3 RESEARCH AREA

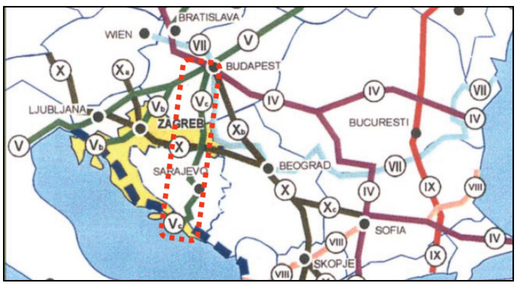

Figure 1 Position of Corridor Vc from Budapest to the Croatian port of Ploče

Corridor Vc is planned to connect the areas south of Budapest, via Osijek in Croatia, with Sarajevo and Mostar in Bosnia and Herzegovina and the Croatian port of Ploce (Fig. 1).

The Corridor Vc in B&H is divided into 13 sections with the total length of 334 km. For the works on the project for the construction of Corridor Vc motorway, section: Počitelj-Bijača, subsection Zvirovići-Bijača, a geodetic network was established designed for 1D mark points of the structure, geometry control during construction and geometry control of the performed works. 1D network should meet several purposes, so the points of the network are stabilised at a sufficient distance from the structure so they are not damaged and their stability is not threatened.

4 PROCESSING OF RESULTS

Traverses of the 1D network are designed in the form of four polygons of geometric levelling (G1, G2, G3 and G4) and three polygons of trigonometric levelling (T1, T2 and T3) (Fig. 2.) 1D network is a geodetic basis for marking and monitoring the structure subsidence in 1D plane and it is stabilised on the foundations of reinforced concrete columns.

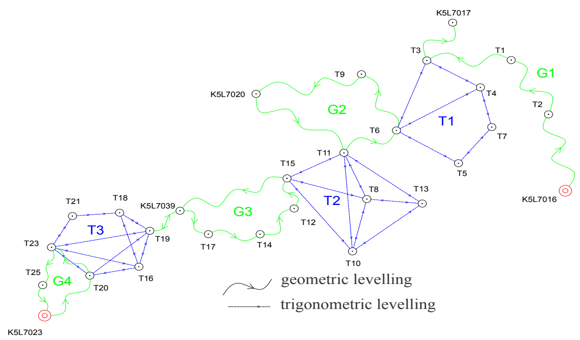

Figure 2 Diagram of measured values

4.1 MEASUREMENT PLAN

In accordance with the parameters obtained from the a-priori assessment of accuracy, measurement is planned in two parts. The measurement plan in the 1D network involves measurement of geometric and trigonometric levelling with the connection of polygons.

In the part of the polygon of geometric levelling, measurement was conducted with LEICA DNA03 digital level, which has the declared measurement accuracy of 0.3 mm/km with invar barcode rods. The level is tested and has a certificate of calibration. Measurements were taken from the middle with the maximum length of the line of sight of 25 m. The conditions during measurement within the geometrical levelling include: before the measurement tempering of instruments and accessories should be performed; during measurement, instrument and tripod should be protected from direct sunlight; measurements should be performed in the period of isotherm, so that the conditions on both rods are about the same; and also the divisions of the rod below 0.50 m and over 2.5 m should not be read. In order to achieve an accuracy of 0.3 mm/km, the measurement procedure implies the measurement back and forth, from the middle, with the change in the instrument height and with the maximum length of the line of sight of 30 m. Monitoring and control of measurements were carried out at the station and within the polygon of geometric levelling.

In the trigonometric levelling, the measurement was performed with the total station LEICA TC2003, which has the declared measurement accuracy (0.5")/1 mm + 1 ppm. Measurements were performed among all the points visible from the station that connected polygons of geometrical levelling. The instrument has been tested and has a certificate of calibration. The conditions during the measurement within trigonometric levelling include: before the measurement, the instrument and accessories were subjected to external conditions for about half an hour; during the measurement, the instrument was protected from direct sunlight; measuring instrument height and prism was carried with the tape that had the certificate of calibration. The measurement procedure implies measurements in both positions of telescope with 5 readings each. The measured values are corrected for the impact of atmospheric parameters, curvature of the Earth's surface and the effect of refraction. Transferring heights from benchmarks built in the foundations of reinforced concrete columns to threads for forced centering on the column (h_t), was carried out with the help of geometric levelling.

4.2 CALCULATION OF 1D COORDINATES OF THE LEVELLING NETWORK

The 1D geodetic network is comprised of a total of 28 points of which 23 are stabilised on reinforced metal plates and 5 polygon points. Calculation of 1D coordinates was performed by adjustment according to the method of least squares. The weights of height differences were determined in accordance with the expression (8), for each measured value. The stochastic model is set up for the case when the measured values are stochastically independent, i.e. when all the elements outside the main diagonal covariance matrix of the measured values are zero. Vertical geodetic Datum is defined in the local 1D system with the heights of polygon points K5L7016 and K5L7023 (Fig. 2). The accuracy of determining the coordinate is in the range from 0.09 mm to 0.81 mm and the average accuracy is 0.43 mm. Terms of reference have been required accuracy of 1 mm, which confirms that the procedure described above is achieved requirements accuracy.

5 CONCLUSION

In geodetic practice, for solving the problem of 1D position in engineering and technical fields, the heights are most often determined by the method of geometric and trigonometric levelling. The selection of the measurement method is in the function of the required accuracy and feasibility conditions of suitable methods for realisation of 1D geodetic network. If different measurement methods are used for height differences, the processing of the measured values in the stochastic model involves the use of different expressions for calculating the mean squared error of the measured value that depend on the type and distribution of measurement results as well as the involved corrections in the measurement results. The accuracy of measuring height differences is adjusted with appropriate processing of the measurement results, by entering the required corrections in the results of measurements and thus the most appropriate value for σ_0^2 is selected.

Within the realised 1D polygon, the a-priori accuracy assessment is applied and the selection of instruments and accessories is carried out. Sigma a-priori is $\sigma_0 = 1$ and sigma a-posteriori from performed measurements is $\hat{\sigma}_0$=1.11. Value of the test hypothesis of the global model is 1.24 which compared with the value of the test statistics of the global model for the adopted level of significance 0.05 is 1.62 and presents the adequacy of the model for $\sigma_{GeomLev} = 0.1$ mm $\sigma_{TrigLev} = 0.55$ mm and $\sigma_{Dist} = 0.2$ mm.

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